$$C(\theta_0, \theta_1) = \underset{(\theta_0, \theta_1)}{\arg\min} \sum_{i=1}^{N} (y_i - f(x_i))^2$$

$$\frac{\partial C}{\partial \theta_0} = \sum_{i=1}^{N} 2(y_i - f(x_i)) \frac{\partial f(x_i)}{\partial \theta_0} (-1)$$

$$\frac{\partial f(x_i)}{\partial \theta_0} = \frac{\partial}{\partial \theta_0} \theta_0 + \theta_1 x_i = 1$$

$$\therefore \frac{\partial C}{\partial \theta_0} = \sum_{i=1}^{N} 2(-1)(y_i - f(x_i)) = -2 \sum_{i=1}^{N} (y_i - f(x_i))$$

$$\frac{\partial C}{\partial \theta_1} = \sum_{i=1}^{N} 2(y_i - f(x_i)) \frac{\partial f(x_i)}{\partial \theta_1} (-1)$$

$$\frac{\partial f(x_i)}{\partial \theta_1} = \frac{\partial}{\partial \theta_1} \theta_0 + \theta_1 x_i = x_i$$

$$\therefore \frac{\partial C}{\partial \theta_1} = \sum_{i=1}^{N} 2(-1) x_i (y_i - f(x_i))$$

$$= -2 \sum_{i=1}^{N} x_i (y_i - f(x_i))$$

Set $\frac{\partial C}{\partial \theta_0}$ and $\frac{\partial C}{\partial \theta_1}$ equal to zero to solve for θ_0 and θ_1.

$$\frac{\partial C}{\partial \theta_0} = 0$$

$$\Rightarrow \quad -2 \sum_{i=1}^{N} \left(y_i - f(x_i)\right) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i - \theta_0 - \theta_1 x_i = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} \theta_0 - \sum_{i=1}^{N} \theta_1 x_i = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i - N\theta_0 - \theta_1 \sum_{i=1}^{N} x_i = 0$$

$$\Rightarrow \quad N\theta_0 = \sum_{i=1}^{N} y_i - \theta_1 \sum_{i=1}^{N} x_i$$

$$\Rightarrow \quad \theta_0 = \frac{1}{N} \sum_{i=1}^{N} y_i - \frac{\theta_1}{N} \sum_{i=1}^{N} x_i$$

$$\Rightarrow \quad \theta_0 = \langle y \rangle - \theta_1 \langle x \rangle$$

$$\frac{\partial C}{\partial \theta_1} = 0$$

$$-2 \sum_{i=1}^{N} \left(y_i - f(x_i)\right) x_i = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} \left(y_i - \theta_0 - \theta_1 x_i\right) x_i = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i x_i - \theta_0 x_i - \theta_1 x_i^2 = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i x_i - \left(\langle y \rangle - \theta_1 \langle x \rangle\right) x_i - \theta_1 x_i^2 = 0$$

$$\Rightarrow \quad \sum_{i=1}^{N} y_i x_i - \langle y \rangle x_i + \theta_1 \langle x \rangle x_i - \theta_1 x_i^2 = 0$$

$$\Rightarrow \theta_1 \langle x \rangle \sum_{i=1}^{N} x_i - \theta_1 \sum_{i=1}^{N} x_i^2 = +\langle y \rangle \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} y_i x_i$$

$$\Rightarrow \theta_1 \sum_{i=1}^{N} x_i^2 - \theta_1 \sum_{i=1}^{N} x_i \langle x \rangle = \sum_{i=1}^{N} y_i x_i - \sum_{i=1}^{N} \langle y \rangle x_i$$

$$\Rightarrow \theta_1 \sum_{i=1}^{N} (x_i - \langle x \rangle) x_i = \sum_{i=1}^{N} (y_i - \langle y \rangle) x_i$$

$$\Rightarrow \theta_1 = \frac{\sum_{i=1}^{N} (x_i - \langle x \rangle) x_i}{\sum_{i=1}^{N} (y_i - \langle y \rangle) x_i}$$