

Sep 17, 2026

last lecture: image formation

pinhole camera model:

(*)
$$\begin{aligned} x &= f \frac{X}{Z} \\ y &= f \frac{Y}{Z} \end{aligned}$$

(X, Y, Z)
world point

(x, y)
image point

matrix multiplication

calibration

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \text{3x4} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$\begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$

camera matrix

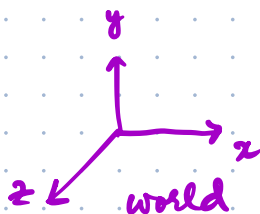
intrinsic matrix

extrinsic matrix

rot. + tra.

(*)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} R & | & t \end{bmatrix}$$

Q. Given a camera rotation matrix: $R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$
plot the camera axes.



Q. Is this a valid camera rotation matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$?

If not how to fix it.

Linear Filtering

filter/kernel: 1 0 2 → not symmetric

Flip conv. (*)

Signal: 2 0 1 2 3 1 0 0 7 8 1

$$\begin{aligned} &\downarrow \\ &(2)(2) + (0)(3) + (1)(1) \\ &= 4 + 0 + 1 \\ &= 5 \end{aligned}$$

5 6 2 7 8 15

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

Convolution is commutative.

$$x[n] * h[n] = h[n] * x[n]$$

Q. Assume that you have some noisy signal:

3 2 1 99 -10 72 0 0 100

How would you remove the noise using an averaging filter?

A. Width of a filter: $2w+1$ half-width

$W=1$,



$W=3$,



Ex: 3-tap averaging filter: $\frac{1}{3}[1, 1, 1] = [\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}]$

Symmetric

cross-correlation vs. convolution

signal: (s) : 2 3 1 1 0 0 9

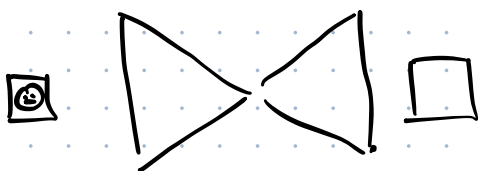
filter: (h) : 1 0 -1



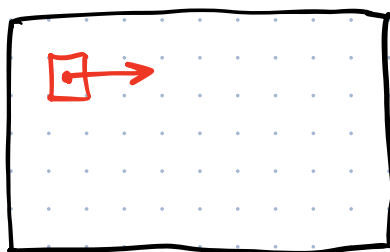
$s * h_1$: -1 ← kernel/filter needs to be flipped

$cc(s, h)$: 1 ← no flip needed.

$$\begin{bmatrix} -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ & -1 & 0 & 1 \\ & & -1 & 0 & 1 \\ & & & -1 & 0 & 1 \\ & & & & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \\ 1 \\ 0 \\ 0 \\ 9 \end{bmatrix}$$



convolution in 2D

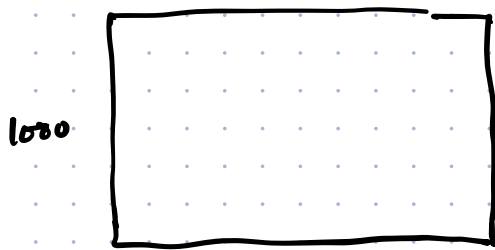


$$* \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

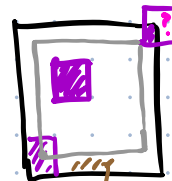
F

$=$

Complexity

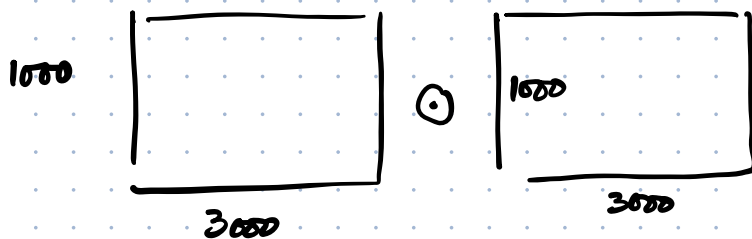


boundary handling.

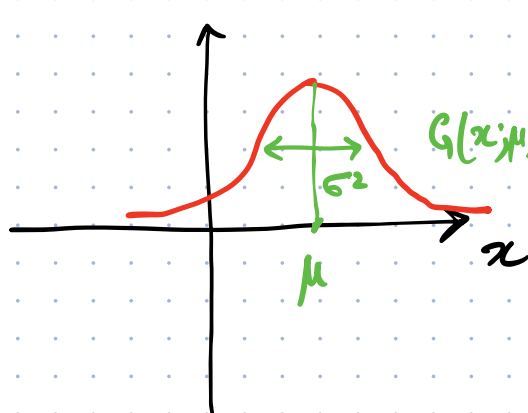
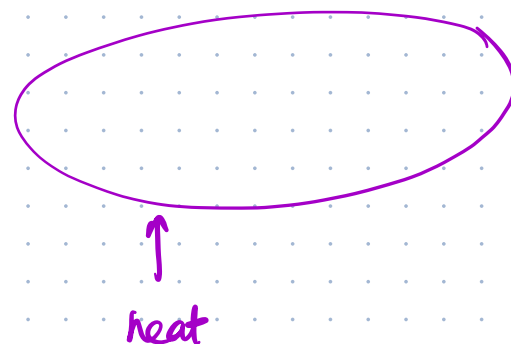


multiplications: $\begin{bmatrix} 3,000,000 \\ \times 15 \times 15 \end{bmatrix}$

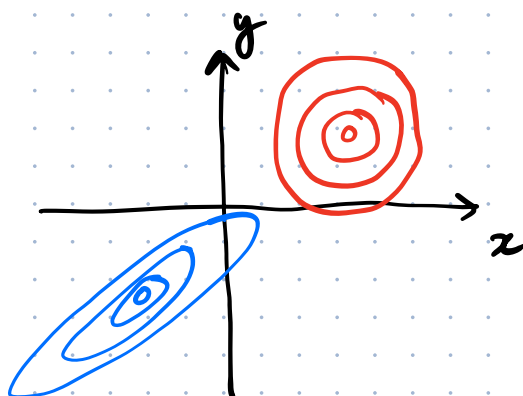
675,000,000 "x"



3,000,000



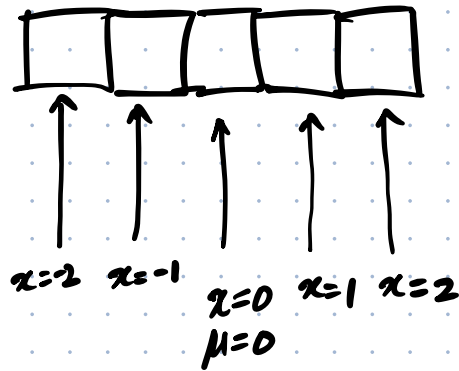
$$G(x, \mu, \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2} (x - \mu)^2}$$



$$G(\mu, \Sigma, x) = \frac{e^{-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)}}{\sqrt{(2\pi)^k \det(\Sigma)}}$$

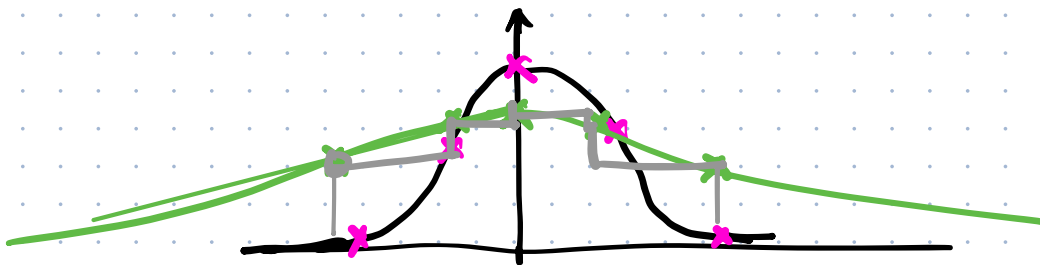
$x = (x, y)$

Q. Construct a 5-tap Gaussian filter?

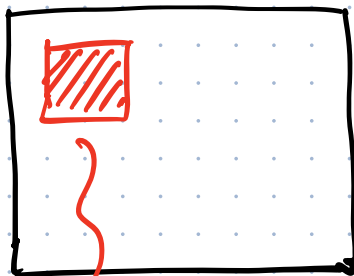


compute: $e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

$\cdot \quad \cdot \quad 1 \quad e^{-\frac{1}{2\sigma^2}} \quad e^{-\frac{4}{2\sigma^2}}$



Separable Filters



1 —

$$\vec{x}^T \vec{x} = \begin{bmatrix} & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

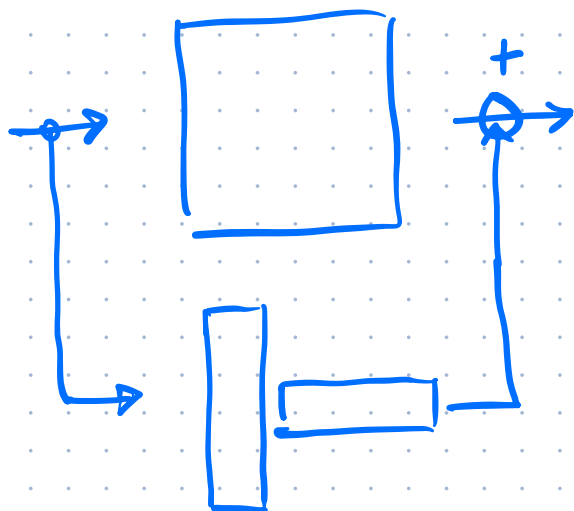
$$\vec{x} \in \mathbb{R}$$

dot product
(inner product)

$$\vec{x} \vec{x}^T = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{bmatrix} & & & \end{bmatrix} = \begin{bmatrix} d \times d \end{bmatrix}$$

outer-product

$$\boxed{} = \sigma_{11} \begin{vmatrix} \end{vmatrix} + \sigma_{22} \begin{vmatrix} \end{vmatrix} + \sigma_{33} \begin{vmatrix} \end{vmatrix} + \dots + \sigma_{kk} \begin{vmatrix} \end{vmatrix}$$



Scale-space analysis

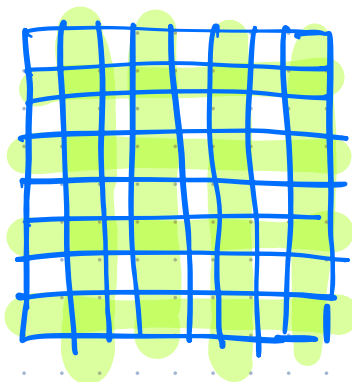


image pyramids
